MATH 120A Prep: Complex Numbers I - Polar Form

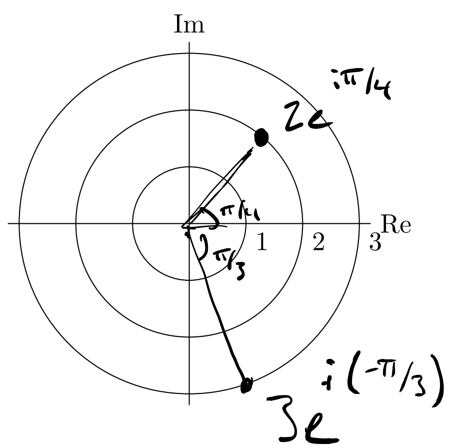
Facts to Know:

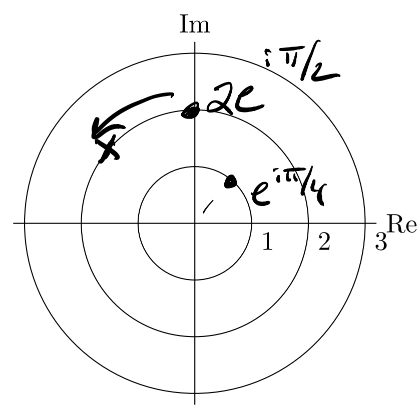
Complex Numbers: $a+bi$ $a, b \in \mathbb{R}$ real numbers $i^2=-1$

- Addition: Adding the real and imaginary parts
- Multiplication: FOIL and simplify using $i^2=-1$

Polar Form: $re^{i\theta}$ r - radius from origin
θ - angle from positive x-axis

- Addition: convert to rectangular form
- Multiplication: $(r_1e^{i\theta_1})(r_2e^{i\theta_2}) = r_1r_2e^{i(\theta_1+\theta_2)}$





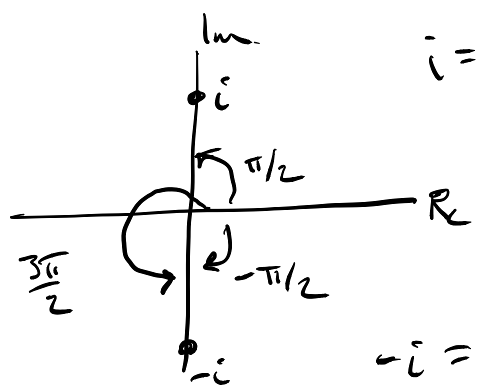
$(2e^{i\pi/2})(e^{i\pi/4})$

$2e^{i(\pi/2+\pi/4)}$

$2e^{i3\pi/4}$

Examples:

1. Write i and $-i$ in polar form.



$i = 1e^{i\pi/2} = e^{i\pi/2}$

$-i = 1e^{i(-\pi/2)}$

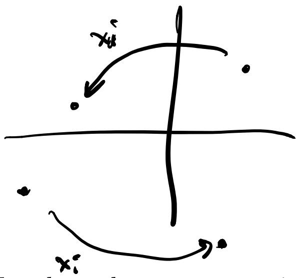
$-i = 1e^{i3\pi/2} = e^{i3\pi/2}$

Note: There are multiple ways to represent a complex number in polar form [add/subtract 2π from the angle]

2. Based on your answer from Question 1, what happens when we multiply by i?

$$i = e^{i\pi/2}$$

$$i \cdot re^{i\theta} = e^{i\pi/2} \cdot re^{i\theta} = re^{i(\theta + \frac{\pi}{2})}$$



same

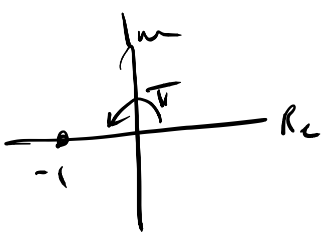
increased by $\pi/2$

rotation by $\pi/2$ radians

3. Use the polar representation of i and -1 to show that $i^2 = -1$.

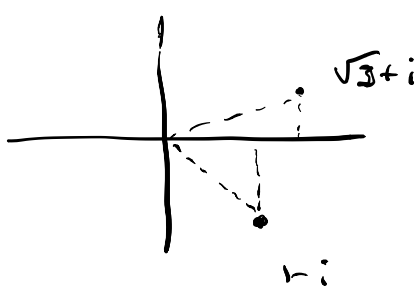
$$i = e^{i\pi/2}$$

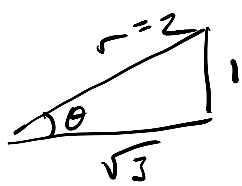
$$i^2 = i \cdot i = e^{i\pi/2} \cdot e^{i\pi/2}$$
$$= e^{i(\pi/2 + \pi/2)}$$
$$= e^{i\pi}$$
$$= -1 \checkmark$$



$$-1 = 1e^{i\pi} = e^{i\pi}$$

4. Write $1-i$ and $\sqrt{3}+i$ in polar form and multiply.

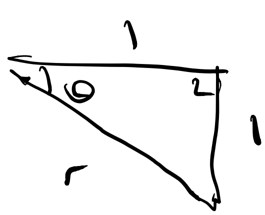




$$r = \sqrt{(\sqrt{3})^2 + 1^2}$$
$$= \sqrt{3+1} = \sqrt{4} = 2$$

$$\tan\theta = \frac{1}{\sqrt{3}} \to \theta = \arctan\left(\frac{1}{\sqrt{3}}\right)$$
$$= \arctan\left(\frac{\sqrt{3}}{3}\right)$$
$$= \pi/6$$

$$\sqrt{3}+i = 2e^{i\pi/6}$$



$$r = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\tan\theta = \frac{1}{1} = 1 \quad \theta = \arctan(1) = \frac{\pi}{4}$$

$$1-i = \sqrt{2}e^{-i\pi/4}$$

$$(2e^{i\pi/6})(\sqrt{2}e^{-i\pi/4}) = 2\sqrt{2}e^{i(\pi/6 - \pi/4)} = \boxed{2\sqrt{2}e^{-i\pi/12}}$$

$$\frac{\pi}{6} - \frac{\pi}{4} = \frac{2\pi}{12} - \frac{3\pi}{12} = \frac{-\pi}{12}$$